FREQUENCY-DEPENDENCE OF THE STATISTICS OF RADIO SOURCES

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Abstract. Without assuming the existence of two populations of radio sources, the counts of vs frequency, spectral index vs flux density, and counts vs spectral index dependences have been derived. An assumption – with arguments to support it – is made that most extragalactic radio sources belong to the class of doubles whose visible parameters depend on their spatial orientation being relative to the observer.

1. Introduction

Through the efforts of many radio astronomers, radio sources have been counted over a wide range of frequencies and fluxes. The results obtained pose a very important question: do the counts at centimeter waves agree with those at low-frequencies, or are the counts affected by evolutionary effects and by new populations of radio sources?

Analysis of the distribution of spectral indices has shown that the number of sources with a flattened spectral increases with higher frequencies (Kellermann *et al.*, 1968). It has been observed, however, that the spectral index of faint radio sources approaches the traditional value – 0.8 when in-depth surveys are made of cm waves (Machalski, 1981; Gorshkov and Konnikova, 1981). It has also been confirmed by the dependence (discussed in Wall *et al.* (1981) and Kellermann and Pauliny-Toth (1981)) between the counts and the spectral indices – i.e., the number of sources with steep spectra becomes greater with weaker flux densities. Some authors explain the possibility of such relationships between the parameters of radio sources by the existence of two populations of objects (steep and flattened spectra) with different cosmological evolutions (Kulkarni, 1978; Wall *et al.*, 1981; Peacock and Gull, 1981). Another, more prosaic explanation may be given to the whole of the observed findings.

Let there be a complete catalog of radio sources up to a flux density $S_{\min}$ at a frequency ν_1. If the distribution of the spectral indices is not a δ-function, the countings curve may be represented by a sum (of curves) $\text{Log}N - \text{Log}S$, each of which involves sources with the same spectral indices. With transition to the frequency ν, each of these curves would move along the axis of the flux densities by the value corresponding to its spectral index $\log S/S_1 = \alpha \log \nu/\nu_1$ without changing its shape. Since, over a wide range of flux densities, the curve $\text{Log}N - \text{Log}S$ is not approximated by a power law, the resulting curve at ν will change its shape as compared with the original. Naturally, it is accompanied by a change in the spectral composition of the radio sources at a fixed flux density S; i.e., the flux density of the sources whose spectra are steep will decrease more rapidly with increasing frequency than that of sources with flattened spectra.

Hence, the fraction of the latter increases with increasing frequency, there appears a *spectral-index vs flux* density dependence and, as a result, a dependence between counts and spectral indices.

Whether the above-mentioned arguments are valid may be checked by modeling radio-source statistics at a single frequency, rescaling it to other frequencies, and comparing the calculated result with the experiment. A Monte Carlo method was applied to the problem which helps to approximate the calculations as much as possible to the experiment and easily find the solutions to a broad range of questions.

Statistics at the frequency 408 MHz were initially selected. At that frequency the $n(S)$-dependence has been studied up to 0.01 Jy (Pooley and Ryle, 1968; Mills *et al.*, 1973), the distribution of spectral indices is known (Ekers, 1969; Maslowski, 1972a, b; Lawrence *et al.*, 1982), and no $\alpha(S)$-dependence has been observed. This last fact is not essential, but it simplifies calculations. Thus, a computer-aided catalog was produced of the sources whose parameters obeyed the statistics at the frequency $\nu_1 = 408$ MHz. At this frequency, only two source parameters are of interest to us, they are the flux density S_1 and the spectral index α_1. Besides, the shape of the source spectrum should be known in order to make the rescaling to frequency ν possible. The statistics were planned to be analyzed over a wide frequency range from 178 to 8700 MHz, although the spectrum approximation by a straight line is not a satisfactory solution.

A more realistic solution was the approximation by a second-order polynomial at a log-scale

$$\log S_\nu = a \log^2 \frac{\nu}{\nu_1} + b \log \frac{\nu}{\nu_1} + c \,. \tag{1}$$

To derive its factors, a third parameter is required along with the flux density S_1 and the spectral index α_1. Thus, a spectral index α_2 of the source at the frequency $V_2 = 8700$ MHz is introduced; and we easily get

$$a = \frac{\alpha_2 - \alpha_1}{2 \log \frac{\nu_2}{\nu_1}} \;; \quad b = \alpha_1 \;; \quad c = \log S_1 \,.$$

The spectral index at the frequency ν is specified as a tangent to the spectrum

$$\alpha_\nu = 2a \log \frac{\nu}{\nu_1} + b \,. \tag{2}$$

Hence, each source from a simulated catalog is characterized by three random independent numbers: S_1, α_1, and α_2.

The quantity S_1 follows the experimental distribution of flux densities at 408 MHz which may be approximated by the function

$$\mathrm{d}n = \rho S^{-2.34 - 0.34 \log S} \, \mathrm{d}S \,. \tag{3}$$

The distribution of the spectral indices at 408 MHz is compiled from the results of the above-mentioned papers and approximated by

$$\phi(\alpha) = 21(\alpha + 1.59)\, e^{-3.24(\alpha + 1.59)^2} . \tag{4}$$

If $\alpha < -1.59$, $\phi(\alpha) \equiv 0$. The mean spectral index of that distribution $\bar{\alpha} = -0.85$ and the fraction of sources with $\alpha > -0.5$ is about 10%.

The α_2-distribution is not yet known as the sensitivity of the surveys made in the shortwave cm range is low (~ 0.3 Jy), and it is mainly the sources with flattened spectra which exceed that level. Regarding most sources with a steep spectrum at low frequencies, they are below the detection threshold and have still to be studied. However, at 1400 MHz (Machalski, 1981), 2700 MHz (Grueff *et al.*, 1980), and 8700 MHz (Gorshkov and Konikova, 1981), and as the results of the Zelenchuk survey at 3900 MHz show, it is evident that the parameters of spectral indice distributions at high frequencies approach the parameters of low-frequency distribution when moving to weaker flux densities. That is why the author has assumed that the true distribution at 8700 MHz repeats that of the spectral indices at 408 MHz. The α_2 distribution was given by the function (4), the only difference being that there is a possibility of a change in its dispersion with the constant $\bar{\alpha}_2 = -0.85$.

The simulated catalog lists 40000 objects with a flux density exceeding 0.01 Jy. The distributions of their spectral indices and flux densities normalized to the density of the sources in the Euclidean model at 408 MHz are shown in Figures 1 and 2.

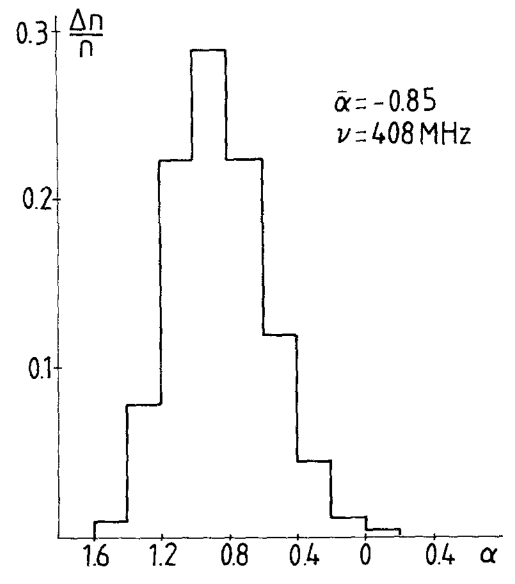

Fig. 1. Distribution of the spectral indices of sources from the simultated catalog.

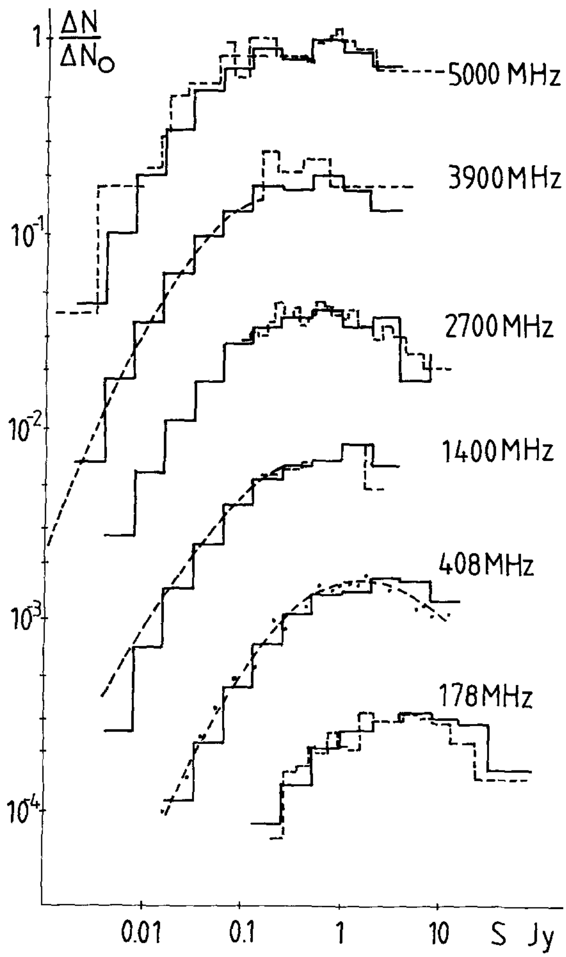

Fig. 2. Experimental and calculated counts of radio sources in the 178 to 5000 MHz range.

2. Calculations of the Statistics of Sources at Frequency ν

The flux density S_ν and the spectral index α_ν of each object from the catalog at ν are determined from (1) and (2). Thus, we get a new list of sources which may be analyzed by traditional methods.

3. Differential Count

The whole flux densities range was subdivided into intervals so that the ratio of their boundaries would be $Si/S_{i+1} = 2$. The number of sources that fall into each interval was normalized to $S_i^{-1.5} - S_{i+1}^{-1.5}$, in proportion with the density of the sources in the Euclidean Universe. The histogram was divided into its maximum to facilitate a comparison with the experiment. The same procedure was used to count the sources with steep ($\alpha < -0.5$) and flattened ($\alpha > -0.5$) spectra.

The *spectral index vs flux* density dependence was calculated by averaging the spectral indices of the sources within each interval.

4. Distribution of Spectral Indices

To see how the distribution parameters depend on the depth of the survey, the process of identifying the sources with the flux density exceeding the threshold was simulated. Then the distribution of the spectral indices related to the identified sources was plotted. At each frequency, calculations were made at three threshold levels $S_{TH} = 0.01$ Jy, $S_{TH} = 0.07$ Jy, $S_{TH} = 0.5$ Jy.

These calculations were made at 178, 408, 1400, 3900, 5000, and 8700 MHz.

5. Comparison of Calculated and Experimental Counts

Figure 2 illustrates the calculated and experimental (dashed) counts of the radio sources at 178, 408, 1400, 2700, 3900, and 5000 MHz. The comparison was based on the experimental works with direct counts of the radio sources: 178 MHz (Gower, 1966;

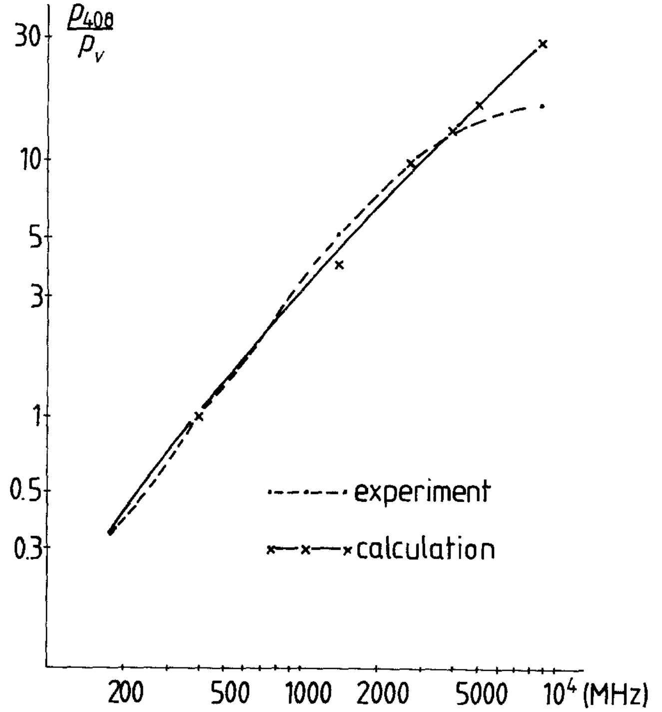

Fig. 3. Experimental and calculated frequency-dependence of radio sources density.

Ryle and Nevill, 1961), 1400 MHz, (Bridl *et al.*, 1972; Maslowski, 1971; Willis *et al.*, 1977), 2700 MHz (the results of Parkes' survey published in Wall *et al.*, 1981), 3900 MHz (Amirkhanyan and Gorshkov, 1983), 5000 MHz (Pauliny-Toth *et al.*, 1972; Bennett *et al.*, 1982).

As the calculations maximally simulate the experiment, one may expect a good agreement not only in the form of counts but also in the source density in the experiment and calculations. Indeed, Figure 3 shows that the frequency-dependence of the density of the sources in both calculations and experiment agree well when the curves were plotted with normalization to the density of sources at 408 MHz. The idyllic picture is, however, somewhat darkened, since the density of source at 8700 MHz has been underestimated by a factor of 1.8. On the whole, the author must state – very surprising as it seems to him – that the calculated and measured counts agree well in both the shape of the curves and the density of sources. Most interesting is the fact that at 3900 and 5000 MHz they still agree, even up to weak flux densities, and no assumption is required about the existence of a population of sources with flattened spectra. Such an assumption was made by Wall (1978) to explain the results of the *P*(*D*) analysis at 2700 and 5000 MHz.

6. Spectral Index – Flux Density

Figure 4 illustrates the experimental and theoretical curves α vs S at 408, 2700, 3900, 5000, and 8700 MHz. The results fit well at 2700 and 8700 MHz where most-representative samples of radio sources with known spectra were used (Wright *et al.*, 1982; Gorshkov and Konnikova, 1981). At 3900 MHz (Amirkhanyan *et al.*, 1982) and 5000 MHz (Pauliny-Toth *et al.*, 1972) the agreement is qualitative, maybe because of the incomplete samples taken at these frequencies.

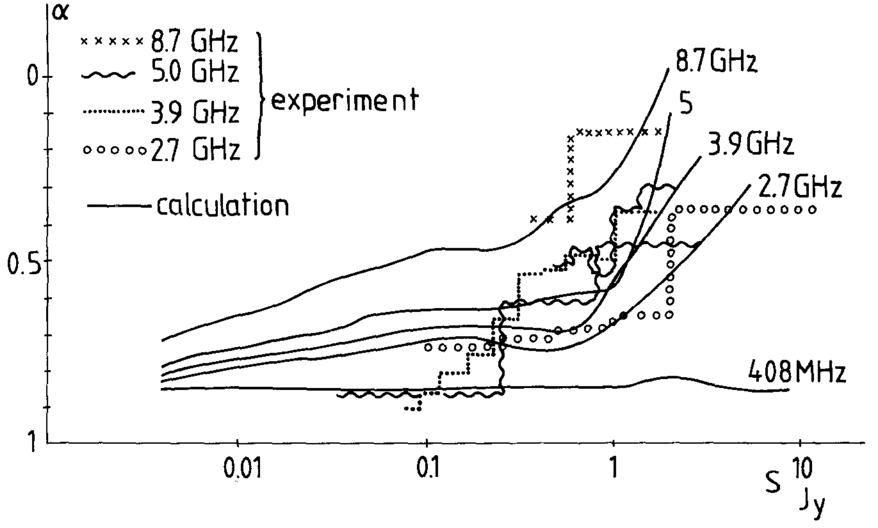

Fig. 4. Experimental and calculated dependence of spectral index vs flux density.

7. Distribution of Spectral Indices

Calculated and experimental distributions at 2700 MHz (Wright *et al.*, 1982), 3900 MHz (Amirkhanyan *et al.*, 1982), and 8700 MHz (Gorshkov and Konnikova, 1981) are compared in Figure 5. Distribution parameters are given there too; to compare the histograms they were normalized to the number of objects in each of them.

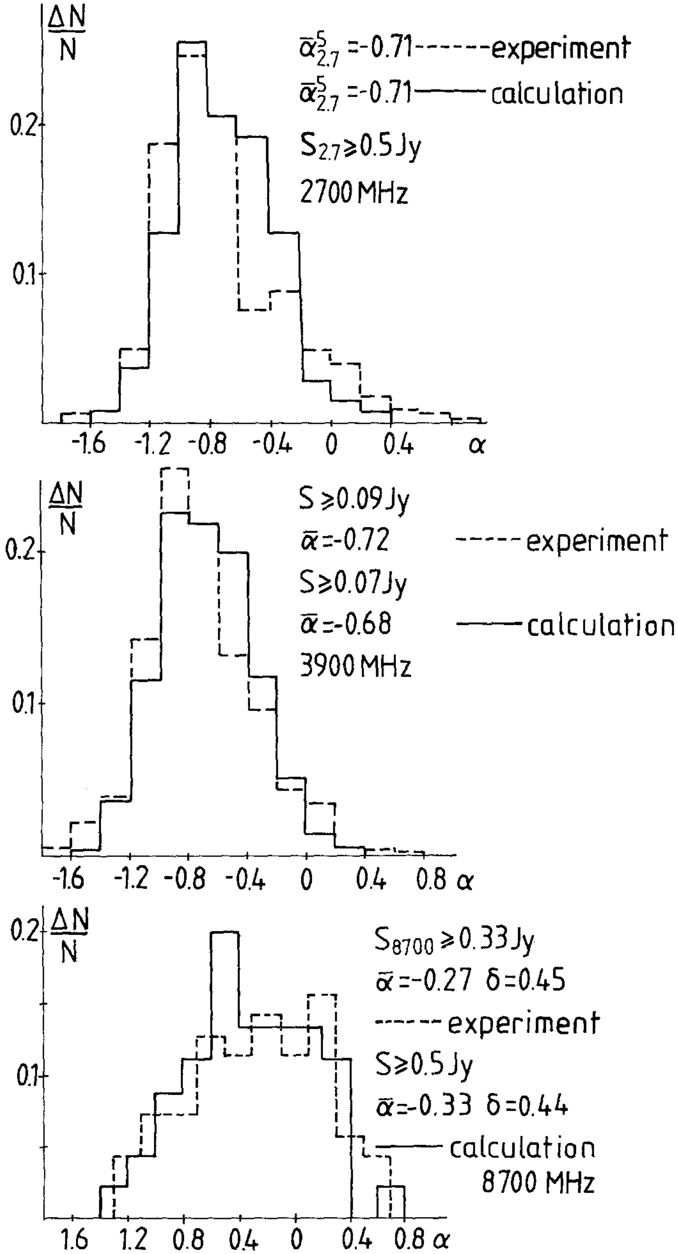

Fig. 5. Experimental and calculated distributions of spectral indices.

8. Counts of Sources with Steep and Flattened Spectra

Calculations and experiments are compared for 2700 MHz (Wall *et al.*, 1981), 5000 MHz (Kellermann and Pauliny-Toth, 1981), 3900 and 8700 MHz (the results of the Zelenchuk survey, 1980–1982). Figure 6 shows that in the experiment and in the calculations toward weaker fluxes, the density of sources with $\alpha > -0.5$ grows slower than with $\alpha < -0.5$. In agreement also the calculated and experimental ratio of densities of these sources in the considered frequency range. The number of sources with flattened spectra is obviously growing toward higher frequencies. To emphasize that this effect does exist in reality, the author demonstrated the results at 8700 MHz though errors in the experimental data are great, due to insufficient statistics (128 sources).

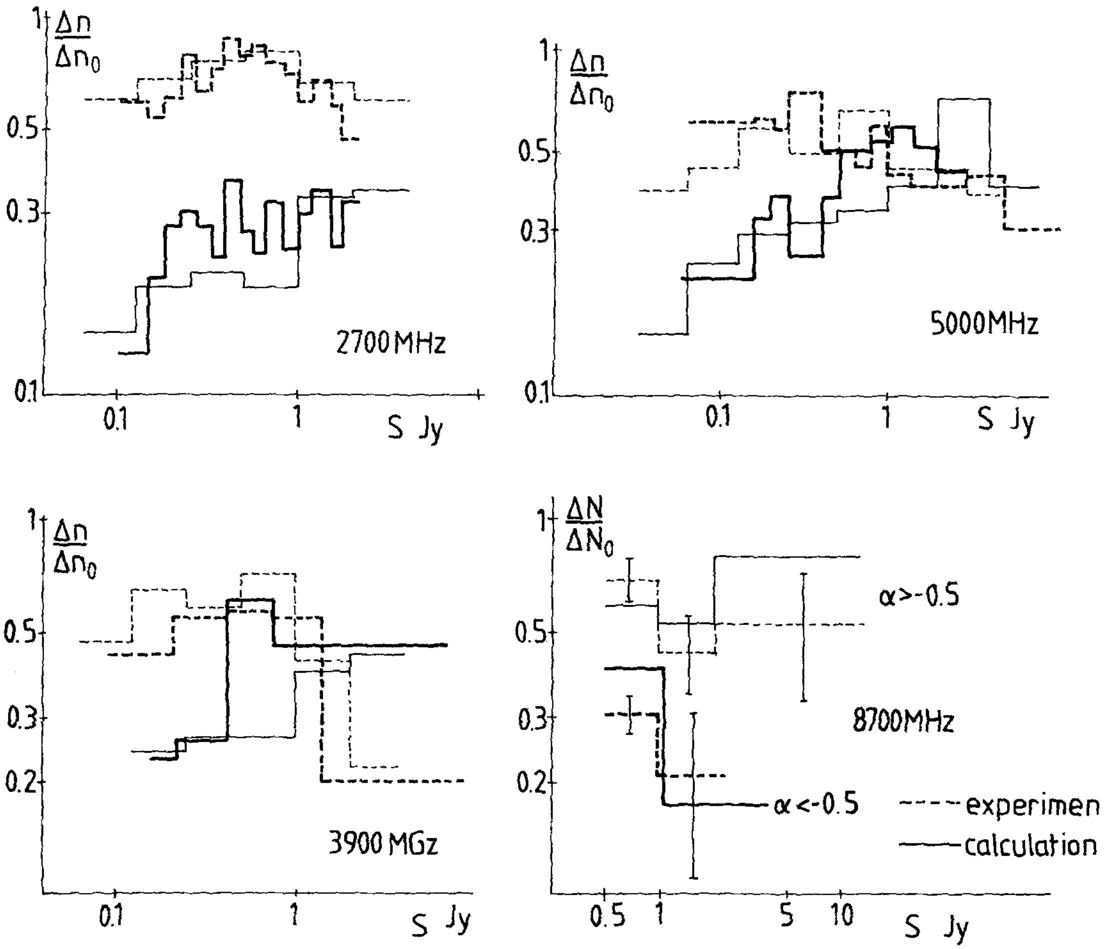

Fig. 6. Counts of radio sources with $\alpha > -0.5$ and $\alpha < -0.5$ (dashed) at 2700, 3900, 5000, and 8700 MHz.
Fine line – calculation.

9. Discussion of Results

Calculations were made using the statistics from the frequency range 178–8700 MHz. All basic statistically-specific features have been derived which, as all mentioned above, implies good agreement with the experiment. This agreement is observed in the shape of the counts in the dependence of the sources' density on the frequency and in the spectral indice distributions. The spectral index vs flux density dependence is derived and counting the sources with steep and flattened spectra agreed with the experiment, without assuming the existence of two populations of radio sources. That means that the evolution of the radio sources – if any exist – is completely given in low-frequency surveys and is one and the same for the sources with steep and flattened spectra, at least in the range of frequencies and flux densities considered. It became possible to get these results by abandoning the idea of power approximation $n(S)$ and taking into account the true distribution of the spectral indices of radio sources.

10. Why are There Only Few Sources with Flattened Spectra?

An assumption suggests itself that there is only one population of radio sources. It is natural for the proponents of two populations to object: how could different values of V/V_{max} for the two groups of sources be explained and how, in general, could one population contain sources with steep and flat spectra that differ so drastically in other parameters as well? The answer to the former may be that, first, the difference, V/V_{max} as now estimated is insignificant and, second, the interpretation of the results is ambiguous. Thus (cf. Masson and Wall, 1977) $V/V_{max} = 0.52 \pm 0.05$ is obtained for 42 sources with $\alpha > -0.5$ and $V/V_{max} = 0.67 \pm 0.05$, for 15 sources with $\alpha < -0.5$. The difference is not significant, as was admitted by the authors of that article. On the other hand, Hawkins and Stewart (1981) showed that such figures may be interpreted as a difference in the luminosity function of sources with different spectra. We should note that separation of the sources into two groups in terms of a spectral index introduces selection in terms of flux density, because an α vs S dependence appears at high frequencies. This very dependence exceeds the ratio of the number of sources with flattened spectra to those with flat spectra. Their ratio is closer to the true one at lower frequencies where α vs S dependence is not observed.

What are the observational characteristics of sources with steep and flattened spectra? Observations of many years and their statistical studies allowed us to draw the following conclusions: steep spectra are usually a property of extended sources with a binary or even more complex structure: the spectral index changes but, weakly over a wide frequency range, the flux density is stable, although some are characterized by the variable emission from the central component and a flattening of its spectrum. Elements of the source are linked by a weak bridge which is sometimes difficult to detect.

Flattened spectra are characteristic of compact sources inclined to variability. VLBI methods could be used to resolve several components in such sources moving at a superlight velocity. Recently, extended haloes with steep spectra have been recorded near such objects (Browne *et al.*, 1982; Soboleva and Temirova, 1983). That certain characteristics of the two types of sources can be united into objects of one type has been frequently suggested (Marscher, 1980; Orr and Browne, 1982; Kellermann and Pauliny-Toth, 1981).

Let a radio source have the following structure. A compact nucleus is linked via regular magnetic fields with outer components whose magnetic fields are entangled. Kovalev and Mikhajlutsa (1980), Kurilchik (1978), and Ozernoy and Ulanovsky (1974) showed that relativistic electrons moving in a regular magnetic field emit, in a cone restricted to a maximum pitch-angle, radiowaves with flattened and inverted spectra. A flare-type radiation from individual clouds is superimposed upon that of the stationary component of an electron flux ($\alpha \geq -0.3$). With the cloud moving from the nucleus, the frequency ν_{max} of the flare-radiation maximum decreases whereas its amplitude changes as $S_{max} \sim \nu_{max}^{0.7}$. It is also shown in those papers that the apparent velocity of a moving cloud $V \sim c \cot\theta$ (where θ is the angle between the magnetic field direction and the line-of-sight) and at small angles it may exceed the velocity of light. It is evident from Kovalev's (1983) paper that an observer will record the flattened spectrum of such a structure if $\theta \leq 10°$. With growing θ, the maximum pitch-angle 'goes away' with the line-of-sight and the radiation intensity of the bridge falls down. Predominant now is the radiation from the outer components with a steep spectrum (e.g., source 3C84 (Berlin *et al.*, 1980) (Figure 7). The steep spectrum of the outer component at low frequencies and the radiation of the bridge, limited from below by the frequency $\nu_l \sim (\sin\theta)^{0.7}$, distinctly manifest here. Then the double structure of a radio source becomes obvious. Variable radiation from the nucleus of an extended radio source is evidently recorded by the observer at the instance when an electron cloud is injected into the regular magnetic field before the pitch angles collapse and are distributed within a solid angle 2π.

Thus, the configuration considered combines the properties of sources with regular and entangled magnetic fields (Shklovskii, 1965) and, depending on its orientation with respect to the observer, may manifest itself either as a compact object with a flattened spectrum or as an extended object with a steep spectrum. Assuming that any spatial position is equally probable and that the observer cannot distinguish θ and $\theta + \pi$

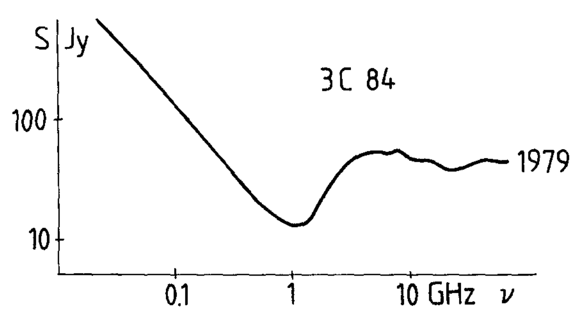

Fig. 7. Spectrum of the 3C84.

orientations, we may easily derive the θ-angle distribution function of the sources to be

$$dP(\theta) = \sin\theta \, d\theta .$$

Hence, the probability of θ being smaller than or equal to θ_{max} is of the form

$$P(\theta_{max}) = 1 - \cos\theta_{max} . \qquad (5)$$

Since no final theory has yet been developed for the source spectral index dependence on its orientation with respect to the observer, we assume that the flattened spectrum ($\alpha \geq -0.3$) is observed if $\theta < \theta_{max} = 10°$. The fraction of such sources is calculated from Equation (5) with the result that $P = 0.015$. In the 408 MHz experiment the fraction of sources with $\alpha \geq -0.3$ is 0.029, which corresponds to $\theta_{max} = 14°$. Since the structure of the source is idealized, the theory that describes it is imperfect although the agreement between the calculations and the experiment seems quite satisfactory. Hence, another argument in favor of the assumption that there exists but one population consisting of double radio sources whose observational characteristics depend on their spatial orientation with respect to the observer.

11. Conclusions

(1) The results of direct counts of radio sources at high frequencies do not point to any excess of weak sources and agree well with low-frequency counts.

(2) Spectral index vs flux density and counts vs spectral index dependences are determined by the $n(S)$ deviation from the power function and by the spectral-index distribution with the dispersion $\sigma > 0$.

(3) The overwhelming majority of extragalactic ratio sources are double and belong to a single population.

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